P 16-3

a) Mean residence time

$$\int_0^{\infty} E(t)\,dt = 1$$

Area under curve

$$A = \frac{\pi \tau^2}{2} = 1 \quad \Rightarrow \tau = \sqrt{\frac{2}{\pi}} = 0.8 \text{ min}$$

For constant volumetric flowrate

$$t_m = \tau = 0.8 \text{ min}$$

b) Variance

$$\sigma^2 = \int_0^{\infty} (t-\tau)^2 E(t)\,dt$$

$$\sigma^2 = \int_0^{\infty} t^2 E(t)\,dt - \tau^2$$

$$\int_0^{\infty} t^2 E(t)\,dt = \int_0^{2\tau} t^2 \underbrace{\sqrt{\tau^2 - (t-\tau)^2}}_{\text{Equation of the } E(t) \text{ curve}}\,dt$$

$$= -\tau^4 \int_{\pi}^{0} \left[\cos^2(x) + 2\cos(x) + 1\right] \sin^2(x)\, dx$$

Use Wolfram alpha!

$$= \frac{5\pi}{8} \tau^4$$

$$\therefore \sigma^2 = \frac{5\pi}{8}\tau^4 - \tau^2 = \frac{1}{2\pi} = 0.159.$$